REPORT 2235-2

REPORT
by
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Investigation of	High Frequency Aircraft Antennas
Subject of Report	The Impedance and Efficiency of Multiturn Loop Antennas
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CHAPTER I
INTRODUCTION

A. Background and Scope

In the HF and lower VHF range of frequencies vehicular antennas, and in particular aircraft antennas, usually are restricted in size to the extent that the largest dimension of the antenna may be much less than a wavelength. Such antennas are said to be "electrically small" although they may actually be relatively large, physically. An electrically small antenna generally tends to have an impedance with a small resistive component due to radiation. If there is loss due to finite conductivity of the structure, the loss resistance may exceed the radiation resistance and the antenna will be relatively inefficient. Impedance and efficiency are important parameters in the design of an antenna system, particularly if substantial power is to be radiated as in a long range communication system.

This study is concerned with the impedance and efficiency of a particular type of HF-VHF antenna, specifically, a multi-turn loop. It should be noted, however, that the techniques discussed here are applicable to any antenna having a well defined set of terminals.

Determination of antenna impedance by experimental methods is straightforward. In the HF-VHF region, impedance bridge and Vector-Volt meter measurements are generally quite satisfactory.* Analytical determination of the antenna impedance is not so straightforward. The radiation resistance may be found by the usual method of closed-surface integration of the far-field Poynting vector[1] if the current distribution on the antenna is known to an adequate approximation. The total impedance is difficult to calculate, generally. The induced EMF method or the near-field Poynting vector integration method[2] may be used for some geometrically simple antennas such as slot, stub, dipole, and single-turn loop antennas. In this study, the far-field Poynting vector integration method is used to find the radiation resistance of a multi-turn loop antenna but experimental methods will be used to determine the total impedance.

Determination of antenna efficiency is a much more difficult problem than determination of antenna impedance. This is particularly true in regard to experimental techniques. The usual method is to obtain both the directivity and absolute gain of the antenna and then obtain the efficiency as the ratio of gain to directivity. This can be done experimentally with good precision at microwave frequencies where the antenna or antenna system can be isolated from the ground

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*These will be discussed in Section III-A.

and other reflecting objects so that accurate patterns can be measured from which to determine directivity, and where the gain of a reference antenna is known accurately enough for the precise determination of gain.

At lower frequencies, and in particular below 100 MHz, several problems arise which make the measurement of antenna efficiency extremely difficult. At these frequencies, one wavelength may be several meters, consequently the antenna (in any practical measurement situation) is relatively close to the ground and other reflecting objects. The effect of the resulting extraneous reflections is to alter the radiation pattern and thereby prevent the accurate determination of antenna directivity.

The same problem occurs when one attempts to measure antenna gain. Gain is usually measured by comparison with a standard-gain antenna, and since the standard-gain antenna is usually geometrically different from the antenna to be measured, the effect of extraneous reflections is to alter the patterns of the standard and test antennas in different ways, thus destroying the accuracy of the gain measurement.

Another problem in the measurement of antenna efficiency is the relatively low efficiency and poor impedance matching of antennas of restricted physical size. This is often the case at frequencies below 100 MHz. If the efficiency of the antenna is low and the antenna is

poorly matched to its feed system, then the effects of radiation from cables and the antenna feed structure may become significant. This further complicates the measurement of antenna gain and directivity.

The crux of the problem with both gain and directivity measurements is that these measurements depend entirely on the measurement of antenna field intensity. It would be very useful to have an efficiency measurement technique which depends not on the measurement of antenna field intensity but rather on the measurement of antenna terminal impedance, since this is much more straightforward in the HF and the lower VHF region. Such a technique has been developed and is discussed in Section III-B-2.

B. The Basic Problem

In practice the efficiency of an antenna generally decreases as the electrical size of the antenna is decreased. This is related to impedance behavior in that at frequencies below the first resonance the radiation resistance generally decreases with antenna size. This study is concerned with the efficiency and the input impedance of a multi-turn loop antenna having small enclosed area.

The behavior of a small circular loop with constant current has been described by many authors such as Foster[3] and Kraus.[1] But the analyses have been restricted to loops which are small in terms

of wavelength and/or loops of only one turn. For loops of larger size, more sophisticated analyses involving integral equations for the currents on the loop have been given by Harrington[4] and Storer;[5] but, again, these analyses have been restricted to loops of only a single turn. The question of radiation efficiency has been ignored by the above-mentioned authors.

The purpose of the present work is to verify, experimentally, the predictions of a theoretical calculation of the radiation efficiency and input resistance of a multi-turn loop antenna at frequencies below 150 MHz.

C. Illustration of the Type of Loop to be Considered

This study is concerned with a class of multi-turn loop antennas for which the circumference of a single turn is small in terms of wavelength, but with a sufficient number of turns such that the total wire length may be several wavelengths.

Figure 1 is a diagram of the geometry of the loop to be studied. The antenna consists of N circular turns of wire, each turn of radius a. The conductor is of circular cross section with radius b. The adjacent turns of the loop are separated by a distance 2d.

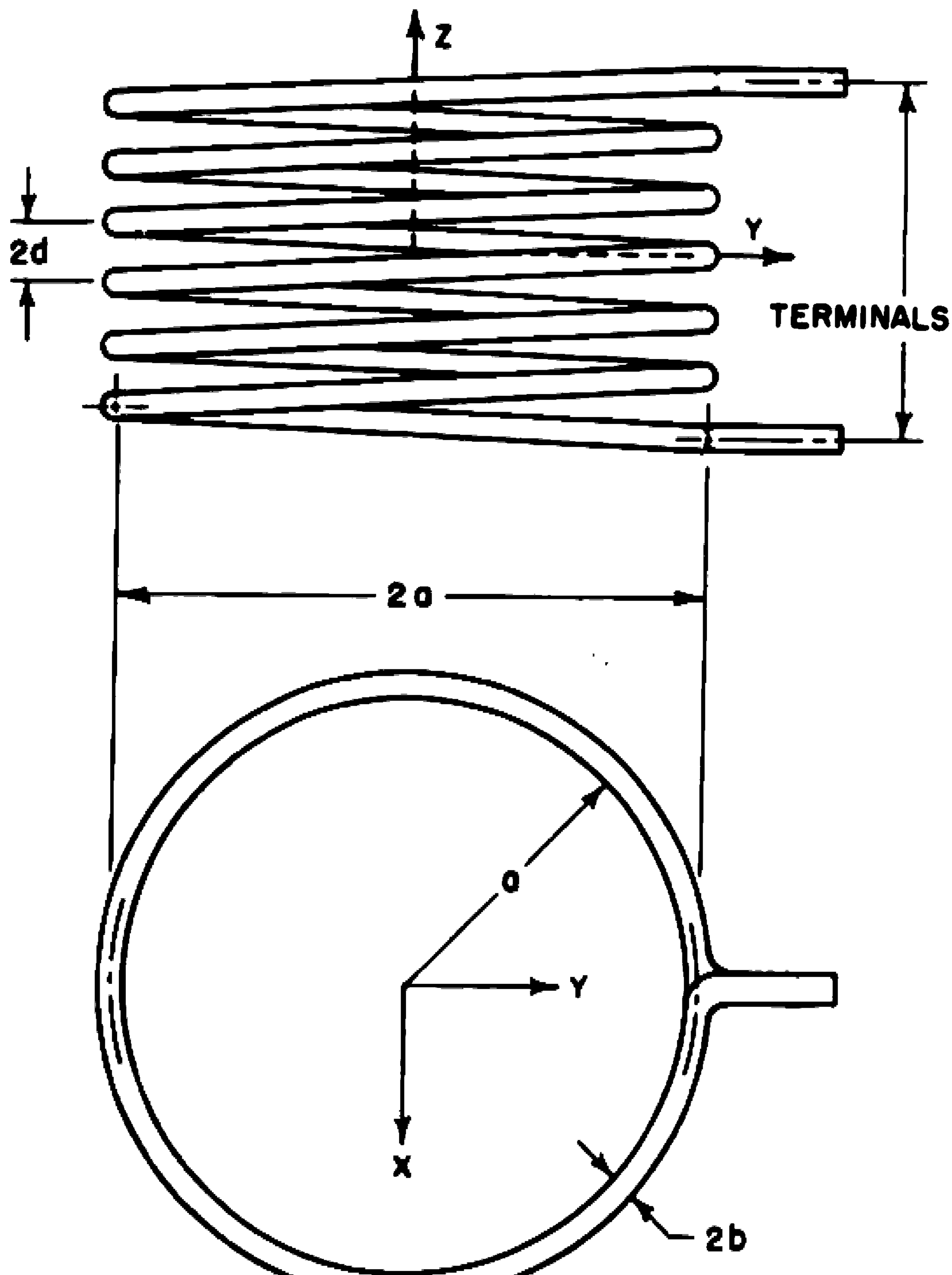

Fig. 1. Geometry of a multi-turn loop antenna.

As mentioned above, the radius a is chosen so that the circumference of a single turn is small in comparison with wavelength, i. e.,

(1) $$2\pi a << \lambda .$$

The number of turns may be such that the total wire length $2\pi Na$ is not necessarily small in terms of wavelength. It is also assumed that $d << a$.

A standard model is chosen so that theoretical and measured results may be compared. The parameters of the model are as follows:

(2) $$\begin{cases} N = 5 \text{ turns} \\ a = 0.2 \text{ meters} \\ b = 0.000794 \text{ meters} \\ d = 0.005 \text{ meters} \end{cases}$$

Calculated and measured values of impedance and efficiency are to be compared for the antenna specified in (2).

CHAPTER II
THEORETICAL DEVELOPMENT

A. Input Resistance

B. A. Munk of The Ohio State University ElectroScience Laboratory has recently completed an analysis of the multi-turn loop antenna which yields the radiation resistance and loss resistance at the terminals of the loop.[6] A brief outline of Munk's analysis is given below.

The current on the loop is assumed to be of the form

$$\overline{I}(\phi') = \hat{\phi}' I_o \cos k_o a\, \phi' \tag{3}$$

where $k_o = \omega \sqrt{\mu_o \epsilon_o}$.

The geometry is illustrated in Fig. 2.

The magnetic far-field of a circular source is given by[7]

$$\overline{H} = \frac{jk_o a}{4\pi r}\, e^{-jk_o r} \int_{-N\pi}^{N\pi} \overline{I}(\phi') \times \hat{r}'\, e^{jk_o a \sin\theta \cos(\phi - \phi')}\, d\phi' \tag{4}$$

and the components of the electric field are given by

$$\begin{cases} E_\theta = \dfrac{k_o}{\omega \epsilon_o} H_\phi \\[2ex] E_\phi = \dfrac{-k_o}{\omega \epsilon_o} H_\theta \end{cases} \tag{5}$$

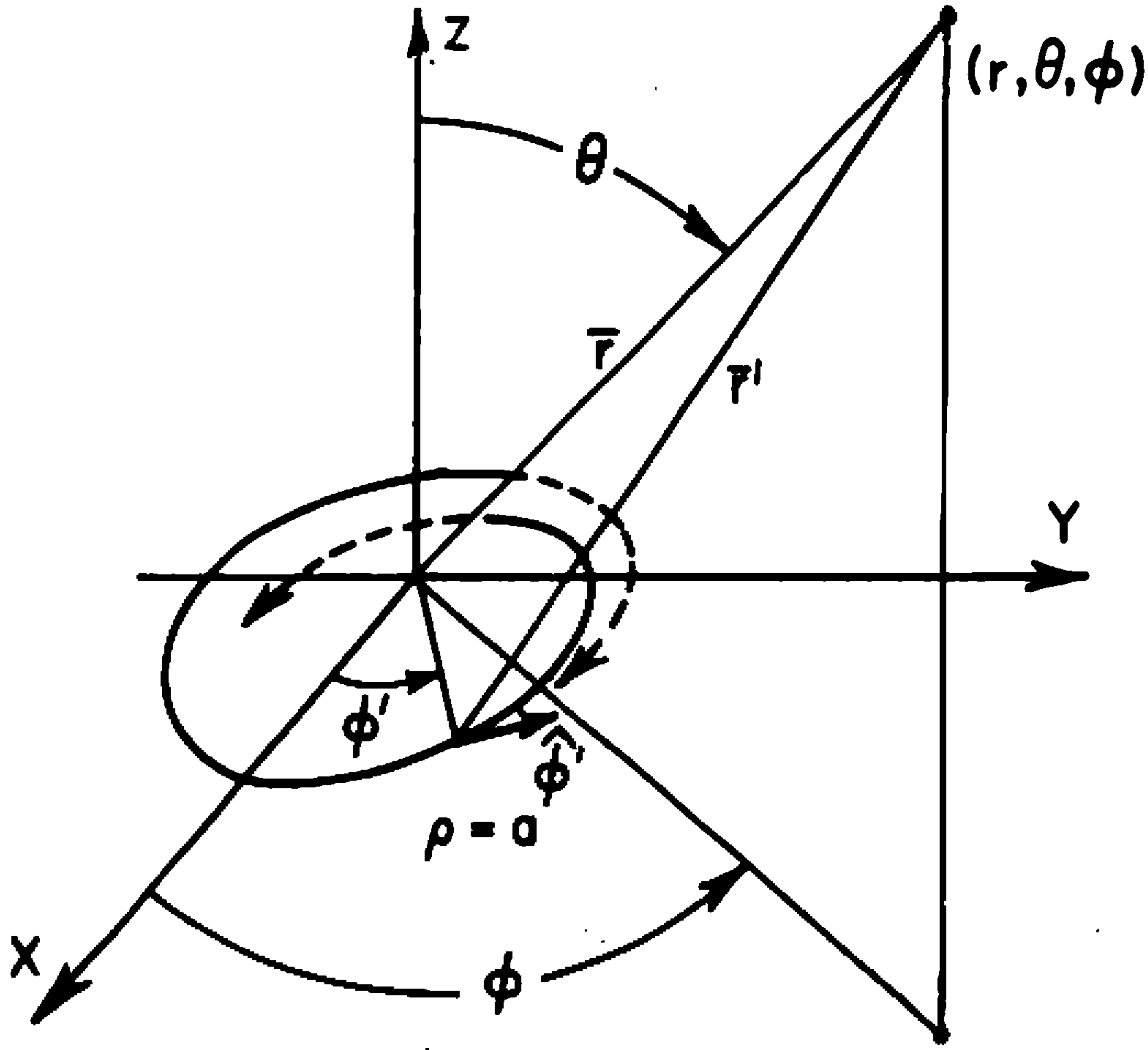

Fig. 2. Coordinate system for Munk's analysis of the multi-turn loop.

By using Eq. (3) in Eq. (4) and inserting the result into Eq. (5), Munk obtains the components of the electric field as:

$$(6) \qquad E_\theta = \frac{j\omega\mu_o a\, e^{-jk_o r}\, I_o}{8r} \cos\theta \, \frac{\sin(N\pi k_o a)}{\sin(\pi k_o a)}$$

$$\times \left[e^{jk_o a(\phi + \frac{\pi}{2})} \left(J_{k_o a+1}(z) + J_{k_o a-1}(z) \right) + e^{-jk_o a(\phi + \frac{\pi}{2})} \left(J_{k_o a+1}(-z) + J_{k_o a-1}(-z) \right) \right]$$

and

(7) $$E_\phi = -\frac{\omega\mu_o a I_o e^{-jk_o r}}{4r}\frac{\sin(N\pi k_o a)}{\sin(\pi k_o a)} \times \left[e^{jk_o a(\phi+\frac{\pi}{2})} \mathbf{J}'_{k_o a}(z) + e^{-jk_o a(\phi+\frac{\pi}{2})} \mathbf{J}'_{-k_o a}(z) \right]$$

where $z = k_o a \sin\theta$ and $\mathbf{J}_\alpha(\beta)$ is the Anger function[8] of order α with argument β.

The radiation resistance R_R of the N-turn loop is calculated from (6) and (7) by the standard technique of Poynting Vector integration in the far-field.[1] The radiated power P_R is

(8) $$P_R = \tfrac{1}{2} R_R I^2_{Terminal} = \int_0^{2\pi}\int_0^{\pi} \frac{|E|^2}{2z_o} r^2 \sin\theta d\theta d\phi \quad .$$

With simplifying approximations, the integral in Eq. (8) is evaluated giving

(9) $$R_R = \frac{8\pi^2 F^2 a^2}{9 \times 10^3} \tan^2(N\pi k_o a) \ ,$$

where F is the frequency in MHz.

The power lost in the antenna, P_L, and the loss resistance, R_L, are related to the current on the antenna as follows:

(10) $$P_L = \tfrac{1}{2} R_L I^2_{Terminal} = \tfrac{1}{2}\int_{-N\pi}^{N\pi} \frac{R_s}{2\pi b} I^2(\phi')\, a d\phi' \ ,$$

where R_s is the surface resistivity of the copper wire given by[9]

(11) $$R_s = 2.16 \times 10^{-4} \sqrt{F} \ \Omega \quad .$$

In evaluating (10), Munk obtains

$$(12) \qquad R_L = \frac{4.16 \times 10^{-5}}{b} \; \frac{N\pi a \sqrt{F}}{\cos^2(N\pi k_o a)} \left[1 + \frac{\sin(2N\pi k_o a)}{2N\pi k_o a}\right] .$$

For the model conforming to the conditions given in (2), the values of R_R and R_L have been calculated in the band from 1 MHz to 150 MHz. The results are shown plotted in Fig. (3). The terminal input resistance R is given by

$$(13) \qquad R = R_R + R_L \quad ,$$

and is plotted in Fig. (4).

As shown in Fig. (4), Munk's analysis predicts sharp peaks in the input resistance at frequencies where the total wire-length is an odd number of half-wavelengths. This prediction is consistant with the known behavior of loop antennas of this type. *

B. Efficiency of a Multi-Turn Loop

The radiation efficiency of an antenna is customarily defined as[10]

$$(14) \qquad E = \frac{P_R}{P_R + P_L} \quad .$$

*This statement will be verified in section IV-A where measured results are presented.

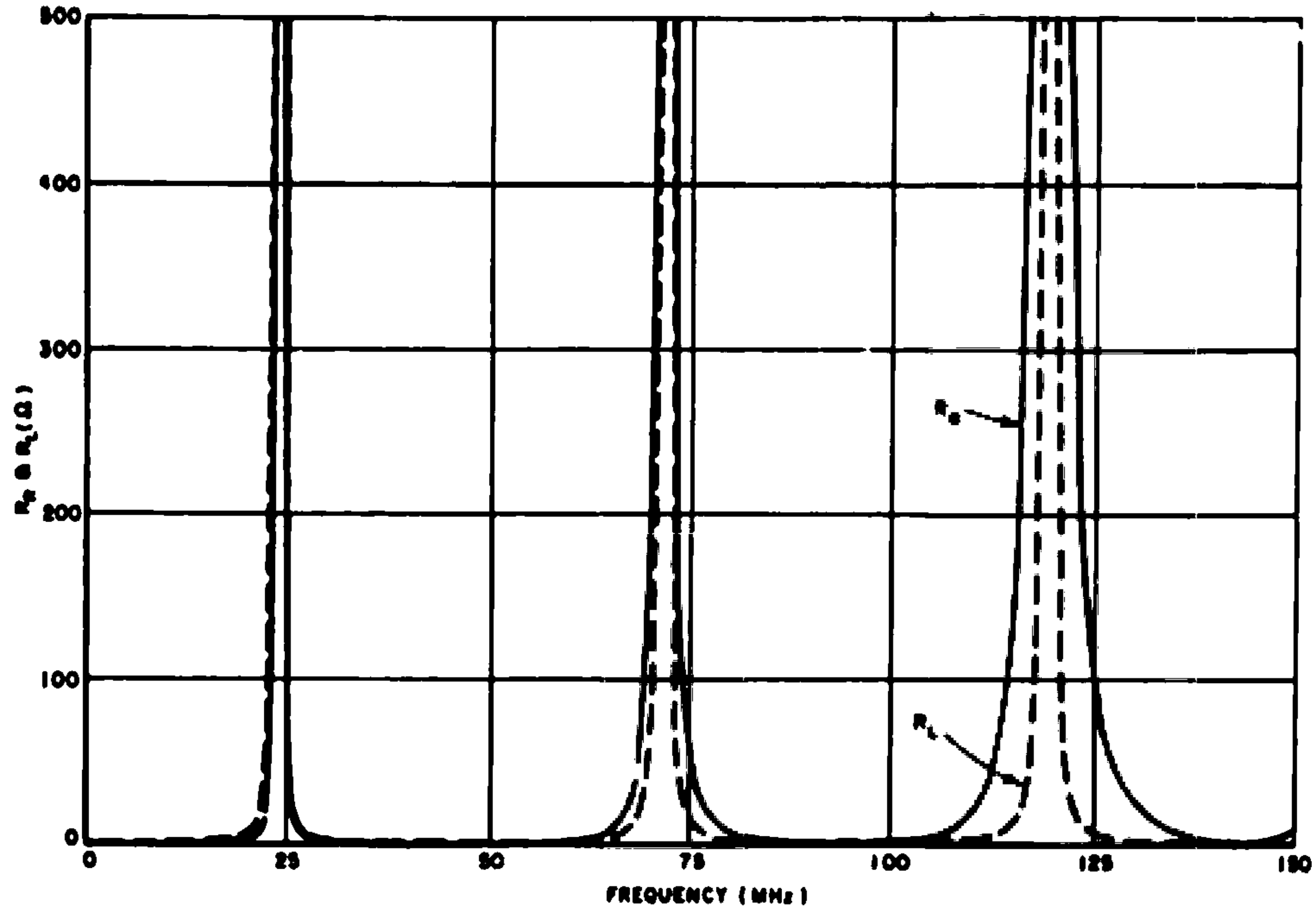

Fig. 3. Radiation resistance and loss resistance of a 5-turn loop (Munk' s formulas).

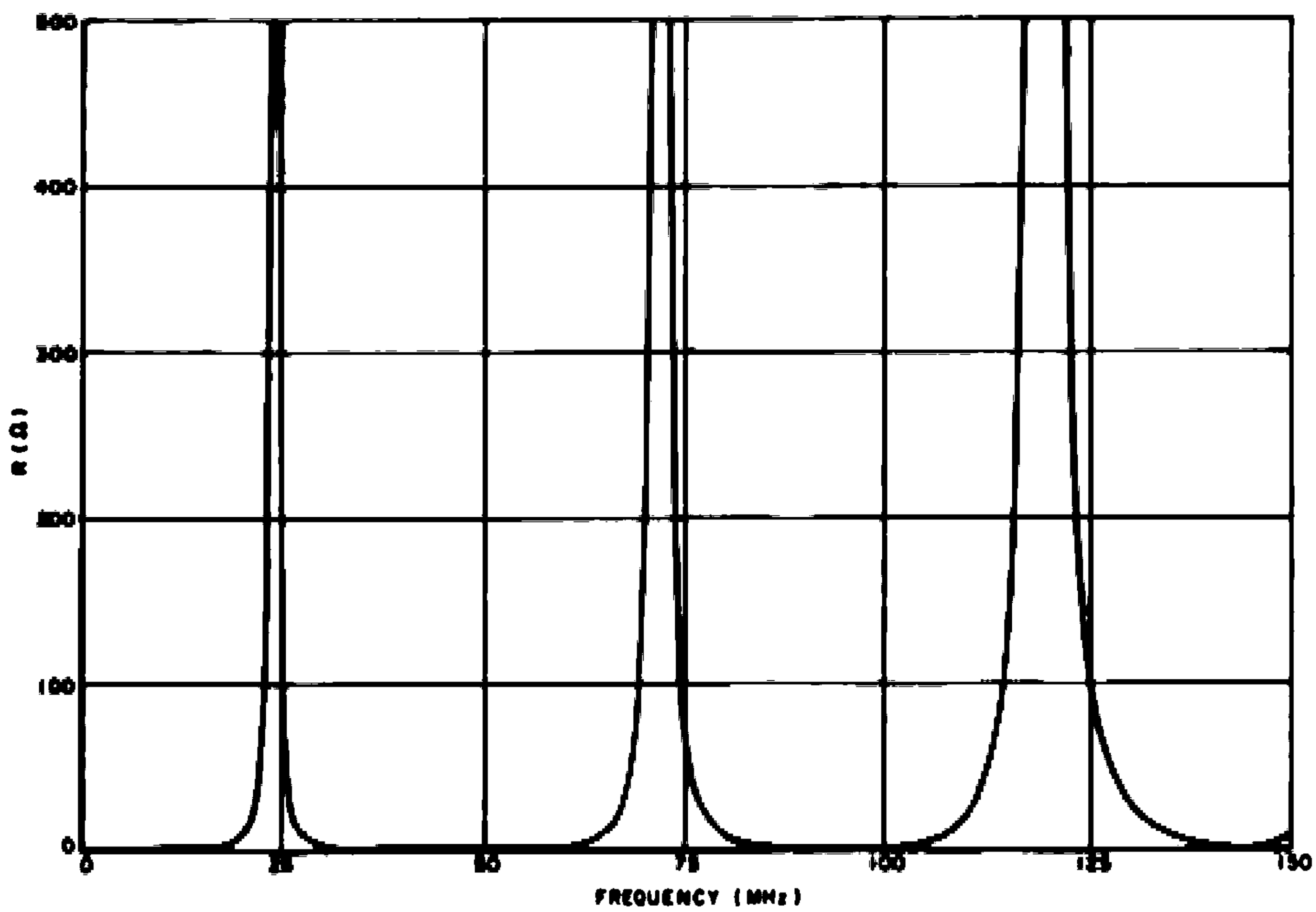

Fig. 4. Input resistance of a 5-turn loop (From Munk' s formulas).

Using Eq. (8) and Eq. (10) in Eq. (14) gives

$$E = \frac{\frac{1}{2} R_R I^2_{\text{Terminal}}}{\frac{1}{2} R_R I^2_{\text{Terminal}} + \frac{1}{2} R_L I^2_{\text{Terminal}}}$$

or,

$$(15) \qquad E = \frac{R_R}{R_R + R_L} \quad .$$

When Eq. (9) and Eq. (12) are substituted into Eq. (15), the efficiency of a copper wire antenna is obtained:

$$(16) \qquad E = \frac{\sin^2 (N\pi a k_o)}{\sin^2 (N\pi a k_o) + \dfrac{0.374\ N}{8\ \pi a b F^{3/2}} \left[1 + \dfrac{\sin(2Nk_o\pi a)}{(2Nk_o\pi a)}\right]} \quad .$$

The efficiency of the standard model described in I-C, as calculated from Eq. (16), is shown plotted in Fig. 5.

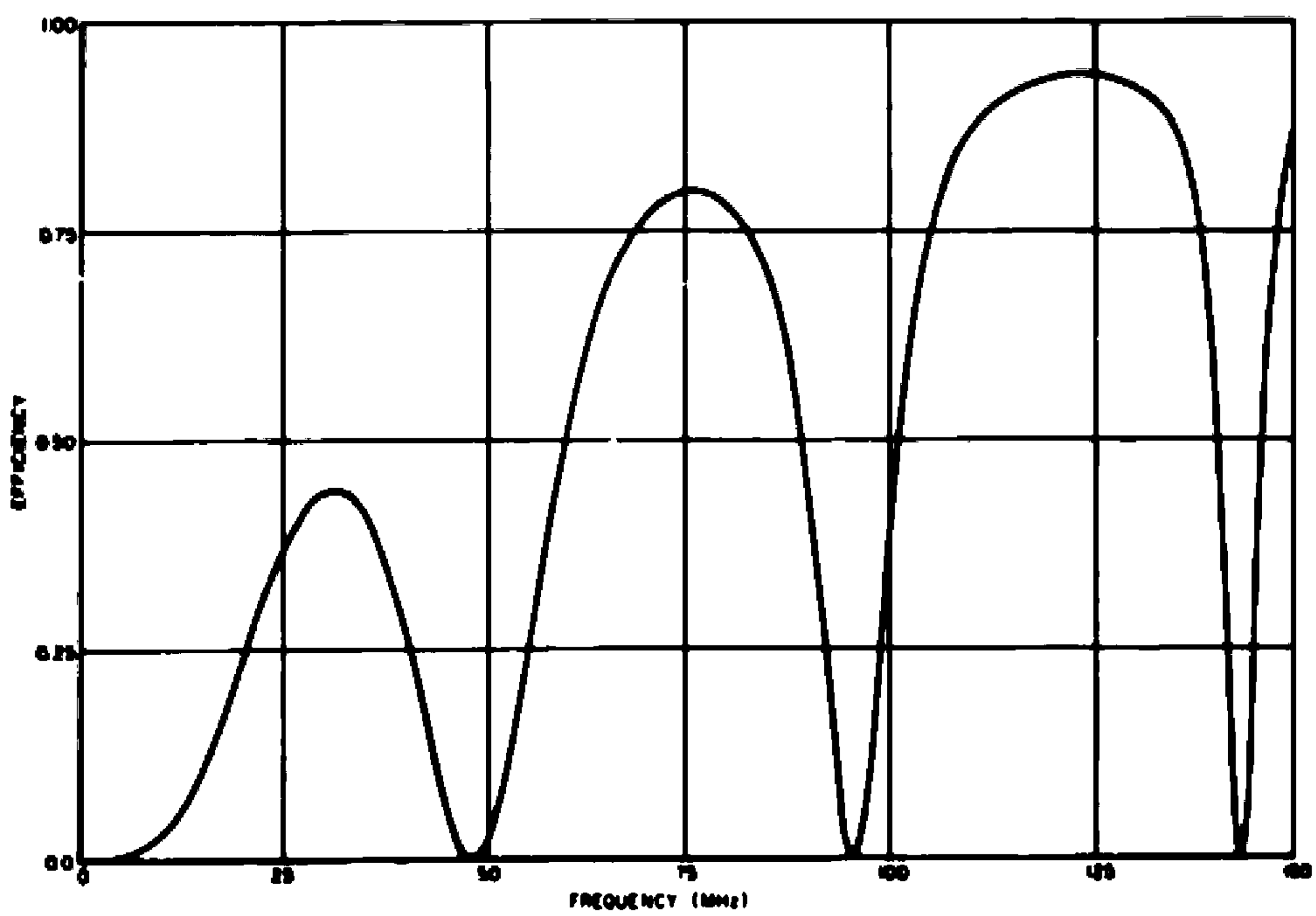

Fig. 5. Efficiency of a 5-turn loop (From Munk's formulas).

CHAPTER III
DEVELOPMENT OF MEASUREMENT TECHNIQUES

A. Impedance Measurements

In order to verify the theoretical developments given previously, it is necessary to measure the input impedance of the model described in I-C. While the loop antenna is a balanced structure, most RF impedance bridges are built for unbalanced inputs. A balun transformer is therefore needed if meaningful measurements are to be made. A commercial wide-band balun transformer (Anzac Model XT-617) was found to be acceptable.

If the impedance at the terminals of the loop antenna is to be obtained from the impedance presented at the terminals of the balun transformer, the equivalent circuit of the balun transformer must be known. An experimental study has revealed that the equivalent circuit of the balun transformer is approximated by an ideal 4-to-1 impedance transformer connected to a short section of transmission line. The parameters of the transmission line are as follows:*

* For complete details, see Appendix I.

(17)

Length	=	0.2171	meters
Characteristic Impedance	=	50.66 + j3.80	ohms
Attenuation Constant	=	0.1601	nepers/meter
Phase Velocity	=	3×10^8	meters/second

The equivalent circuit is shown in Fig. 6.

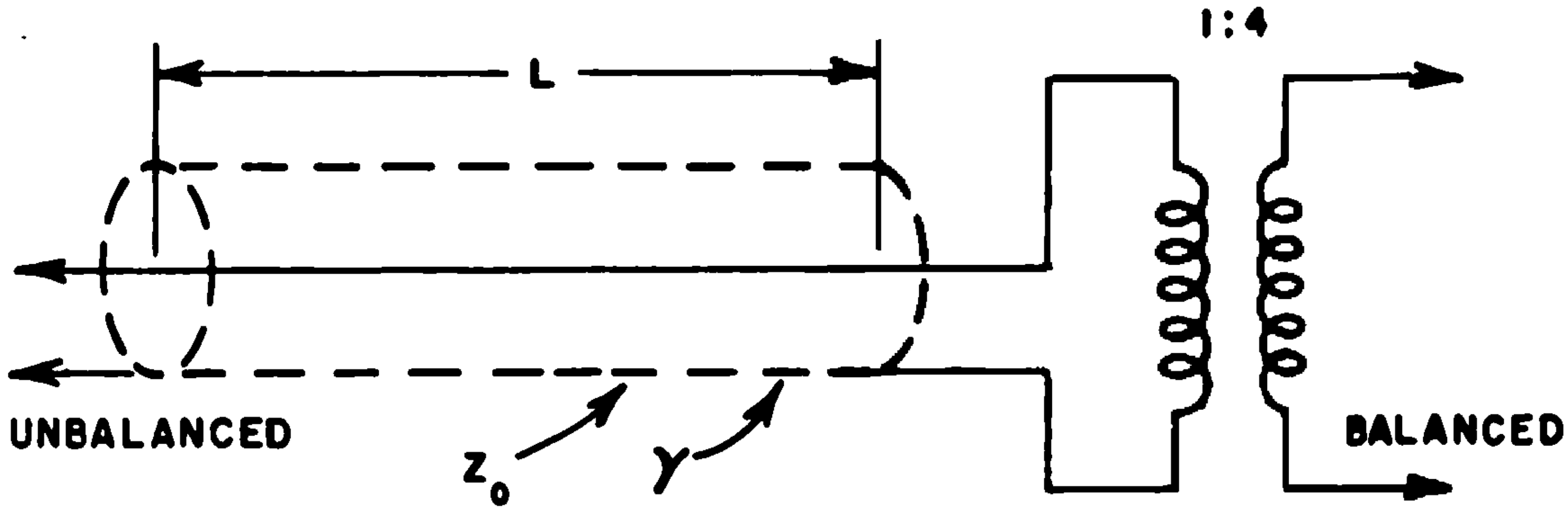

Fig. 6. Equivalent circuit of the balun transformer.

With the parameters of the equivalent circuit and the standard transmission line equations,[11] the terminal impedance of the loop antenna can be calculated from the impedance Z_M measured at the terminals of the balun transformer. The equation for the loop terminal impedance is

$$
(18) \quad \begin{cases} Z = 4 Z_o \dfrac{1 + \Gamma e^{2\gamma L}}{1 - \Gamma e^{2\gamma L}} \\ \text{where} \\ Z_o = 50.66 + j3.80 \text{ ohms} \\ L = 0.2171 \text{ meters} \\ \Gamma = \dfrac{Z_M - Z_o}{Z_M + Z_o} \\ \gamma = \alpha + j \dfrac{2\pi}{\lambda} \quad (\text{meters})^{-1} \\ \alpha = 0.160 \quad (\text{meters})^{-1} \end{cases}
$$

At frequencies above 50 MHz, a Hewlett Packard Model 803A VHF bridge may be used to measure directly the impedance Z_M presented at the balun terminals. At frequencies below the range of the VHF bridge the impedance Z_M may be measured with a Hewlett Packard Model 8405A Vector Voltmeter. A test circuit as shown in Fig. 7 is set up and the voltages V_A and V_B and the phase angle Ψ between them are measured on the Vector Voltmeter.

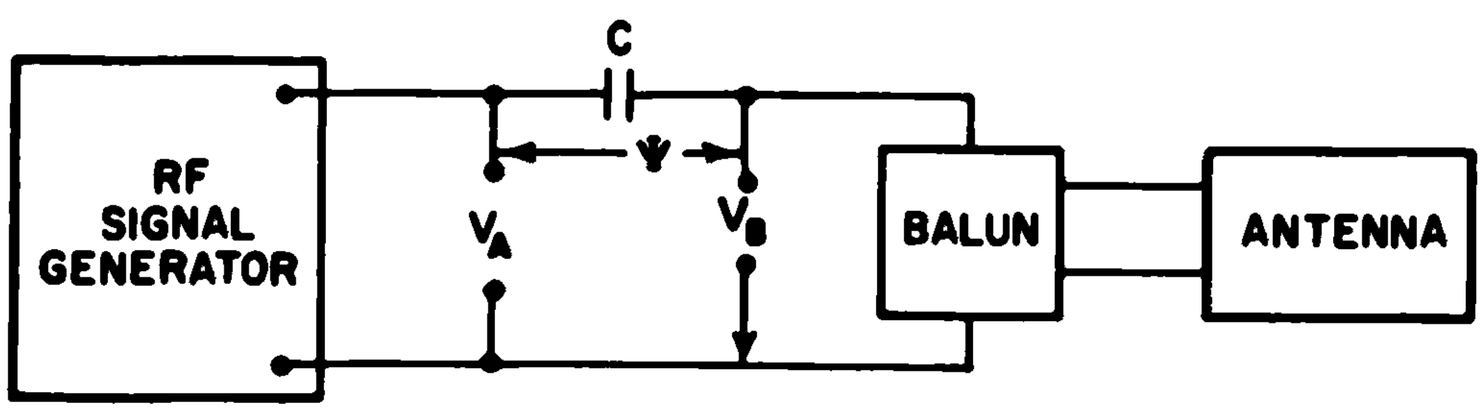

Fig. 7. Test circuit for impedance measurement.

The impedance Z_M is given by

$$(19) \qquad Z_M = \frac{1}{\left(\frac{V_A}{V_B} e^{-j\Psi} - 1\right) j\omega C} \quad .$$

B. Efficiency Measurements

1. Discussion of Established Methods

In the measurement of antenna radiation efficiency, the following relation is most commonly used:[12, 13, 14]

$$(20) \qquad G_o = ED \quad ,$$

where

G_o = absolute gain of the antenna

$= \dfrac{\text{maximum radiation intensity from the antenna}}{\text{radiation intensity from a lossless isotropic source with the same power input}}$,

D = directivity of the antenna

$= \dfrac{\text{maximum radiation intensity}}{\text{average radiation intensity}}$,

and

E = antenna efficiency

From Eq. (20), the antenna efficiency is obtained as

$$(21) \qquad E = \frac{G_o}{D}$$

The usual procedure in efficiency measurements is to obtain D by numerical integration of the far-field pattern function and to measure G_o by comparison with a standard-gain antenna. The

calculation of D is straightforward, though tedious, but two serious problems are encountered when one attempts to measure G_o. One problem arises because the antenna often cannot be removed a sufficient distance from the ground and from supporting structures to prevent reflections from influencing the field intensity. These "site errors" are particularly troublesome in the frequency range below 150 MHz. The other problem arises because the power input to each antenna must be accurately known. Accurate measurement of input power to the antenna may be quite difficult if the frequency is too low to allow the use of a slotted line.

An efficiency measurement technique, which reduces the problem of site-errors, has been developed by T. H. Crowley.[15] The method described by Crowley employs two different antennas which are geometrically identical but constructed of metals with different surface resistivities. With quantities pertaining to antenna No. 1 identified by superscript 1 and quantities pertaining to antenna No. 2 identified by superscript 2, the following equations hold:

$$(\text{power input})^1 = (\text{power radiated})^1 + (\text{power lost})^1$$

or

$$P_I^{(1)} = P_R^{(1)} + P_L^{(1)} \tag{22}$$

and

$$P_I^{(2)} = P_R^{(2)} + P_L^{(2)} \quad . \tag{23}$$

Let $K_R = \dfrac{P_R^{(1)}}{P_R^{(2)}}$, $K_L = \dfrac{P_L^{(1)}}{P_L^{(2)}}$, and $K_I = \dfrac{P_I^{(1)}}{P_I^{(2)}}$.

Crowley gives the efficiency of antenna No. 1 as

(24) $$E^{(1)} = \frac{\dfrac{K_L}{K_I} - 1}{\dfrac{K_L}{K_R} - 1} \quad .$$

If the surface resistivities of the two metals are not too different, the current distributions on the two antennas and the patterns of the two antennas will be very nearly identical, in which case

(25) $$K_L = \frac{P_L^{(1)}}{P_L^{(2)}} \simeq \frac{R_s^{(1)}}{R_s^{(2)}} \quad .$$

Let the surface resistivities of the two metals be related as follows:

(26) $$R_s^{(2)} = r \; R_s^{(1)} \quad .$$

Then if the metals are not too dissimilar. Eq. (25) and Eq. (26) imply that

(27) $$K_L \simeq \frac{1}{r} \quad .$$

Also, the power input to each antenna may be adjusted, by field intensity measurements, so that

(28) $$K_R = \frac{P_R^{(1)}}{P_R^{(2)}} = 1 \quad .$$

With this done, Eq. (24) becomes

(29) $$E^{(1)} = \frac{r - \dfrac{1}{K_I}}{r - 1} \quad .$$

If the two antennas are geometrically identical and the patterns are the same, the problem of site-error is eliminated. The problem of input power measurement is still present; however, only measurement of relative power is required.

Another technique for the measurement of the radiation efficiency of electrically small antennas has been suggested by Wheeler[16]. Wheeler's method makes use of the fact that for an electrically small antenna (assumed centered at the origin of a spherical coordinate system) the near field is to a good approximation confined to the region $r < \frac{\lambda}{2\pi}$, i.e. the interior of the "radiansphere".[16] Thus a conducting sphere placed around the antenna with radius $r \geq \frac{\lambda}{2\pi}$ will reduce the radiation resistance to zero while leaving the loss resistance essentially unchanged. The loss resistance, R_L, may then be measured directly at the antenna terminals. With the conducting sphere removed the radiation resistance plus loss resistance, $R_R + R_L$, is measured at the antenna terminals. The antenna efficiency is then calculated from the equation

$$E = \frac{(R_R + R_L) - R_L}{(R_R + R_L)} \tag{30}$$

Wheeler's method is the simplest and most convenient; but it has a serious limitation at lower frequencies, since the size of the required radiation shield becomes impractically large.

The next few paragraphs describe an efficiency measurement scheme which is practical at low frequencies and which eliminates both the site-error problem and the necessity of making power measurements.

2. A Resistance-Comparison Method

As does Crowley's method, the present scheme employs two antennas which are geometrically identical but materially different. The input resistance, radiation resistance, and loss resistance are related as follows:

$$R^{(1)} = R_R^{(1)} + R_L^{(1)} \quad , \tag{31}$$

$$R^{(2)} = R_R^{(2)} + R_L^{(2)} \quad . \tag{32}$$

Again, if the surface resistivities of the two antennas are not widely different, then the current distributions and pattern functions will be very nearly identical. Therefore to a good approximation

$$R_R^{(1)} = R_R^{(2)} \quad , \tag{33}$$

and

$$\frac{R_L^{(2)}}{R_L^{(1)}} \simeq \frac{R_s^{(2)}}{R_s^{(1)}} = r \quad ,$$

or

$$R_L^{(2)} = r\, R_L^{(1)} \quad . \tag{34}$$

Inserting Eq. (33) and Eq. (34) into Eq. (32) gives

$$(35) \qquad R^{(2)} = R_R^{(1)} + r\, R_L^{(1)} \quad .$$

Multiplying Eq. (31) by r and subtracting Eq. (35) from the result yields

$$r\, R^{(1)} - R^{(2)} = (r-1)\, R_R^{(1)}$$

or

$$(36) \qquad R_R^{(1)} = \frac{r\, R^{(1)} - R^{(2)}}{r - 1} \quad .$$

The efficiency of antenna No. 1 is

$$(37) \qquad E^{(1)} = \frac{R_R^{(1)}}{R^{(1)}} \quad .$$

By combining Eq. (36) and Eq. (37), an expression for the efficiency is obtained:

$$(38) \qquad E^{(1)} = \frac{r\, R^{(1)} - R^{(2)}}{R^{(1)}\, (r-1)} \quad .$$

Eq. (38) shows that the efficiency of an antenna can be calculated when the input resistances of two geometrically identical but materially different antennas are known. The ratio r is easily calculated from the expression[9] for R_s.

$$R_s^{(1)} = \sqrt{\frac{\pi f \mu_1}{\sigma_1}}$$

$$R_s^{(2)} = \sqrt{\frac{\pi f \mu_2}{\sigma_2}}$$

therefore

$$r = \sqrt{\frac{\mu_2\sigma_1}{\mu_1\sigma_2}} \quad . \tag{39}$$

In Eq. (39), μ_1, σ_1 and μ_2, σ_2 are the permeability and conductivity, respectively, of metals 1 and 2.

3. Sensitivity of the Resistance-Comparison Method to Errors of Observation

In order to determine the accuracy of the resistance-comparison method of antenna efficiency measurement, it is necessary to know the error in the measured efficiency as a function of the error in observed quantities. Eq. (38) can be written in the form

$$E^{(1)} = \frac{r}{r-1} - \frac{\dfrac{R^{(2)}}{R^{(1)}}}{r-1} \quad . \tag{40}$$

From Eq. (40) it is seen that $E^{(1)}$ is a function of r and the ratio $\frac{R^{(2)}}{R^{(1)}}$. The error $\Delta E^{(1)}$ in measured efficiency is given in terms of the errors Δr and $\Delta\left(\frac{R^{(2)}}{R^{(1)}}\right)$ in the observed quantities by the equation[17]

$$\Delta E^{(1)} = \frac{\partial E^{(1)}}{\partial r}\,\Delta r + \frac{\partial E^{(1)}}{\partial\left(\frac{R^{(2)}}{R^{(1)}}\right)}\,\Delta\left(\frac{R^{(2)}}{R^{(1)}}\right) \quad . \tag{41}$$

From Eq. (40),

$$
(42)\quad \begin{cases} \dfrac{\partial E^{(1)}}{\partial r} = \dfrac{\dfrac{R^{(2)}}{R^{(1)}} - 1}{(r-1)^2} \\[2ex] \dfrac{\partial E^{(1)}}{\partial\left(\dfrac{R^{(2)}}{R^{(1)}}\right)} = -\dfrac{1}{r-1} \end{cases}
$$

Using Eq. (42) in Eq. (41), the maximum error in efficiency in terms of errors in r and $\dfrac{R^{(2)}}{R^{(1)}}$ is

$$
(43)\qquad \Delta E^{(1)} = \frac{\left(\dfrac{R^{(2)}}{R^{(1)}} - 1\right)\Delta r}{(r-1)^2} - \frac{\Delta\left(\dfrac{R^{(2)}}{R^{(1)}}\right)}{r-1} .
$$

Eq. (43) shows that when the efficiency being measured is high, (i. e. $R^{(2)}/R^{(1)} \simeq 1$) the error contribution from Δr is very small.

CHAPTER IV
MEASURED RESULTS

A. Impedance Measurements

Impedance measurements were taken on the 5-turn loop of Section I-C in the frequency range from 0 to 150 MHz. A computer was used to perform the calculations indicated in Eq. (18) and the results are shown plotted in Figs. (8) and (9).

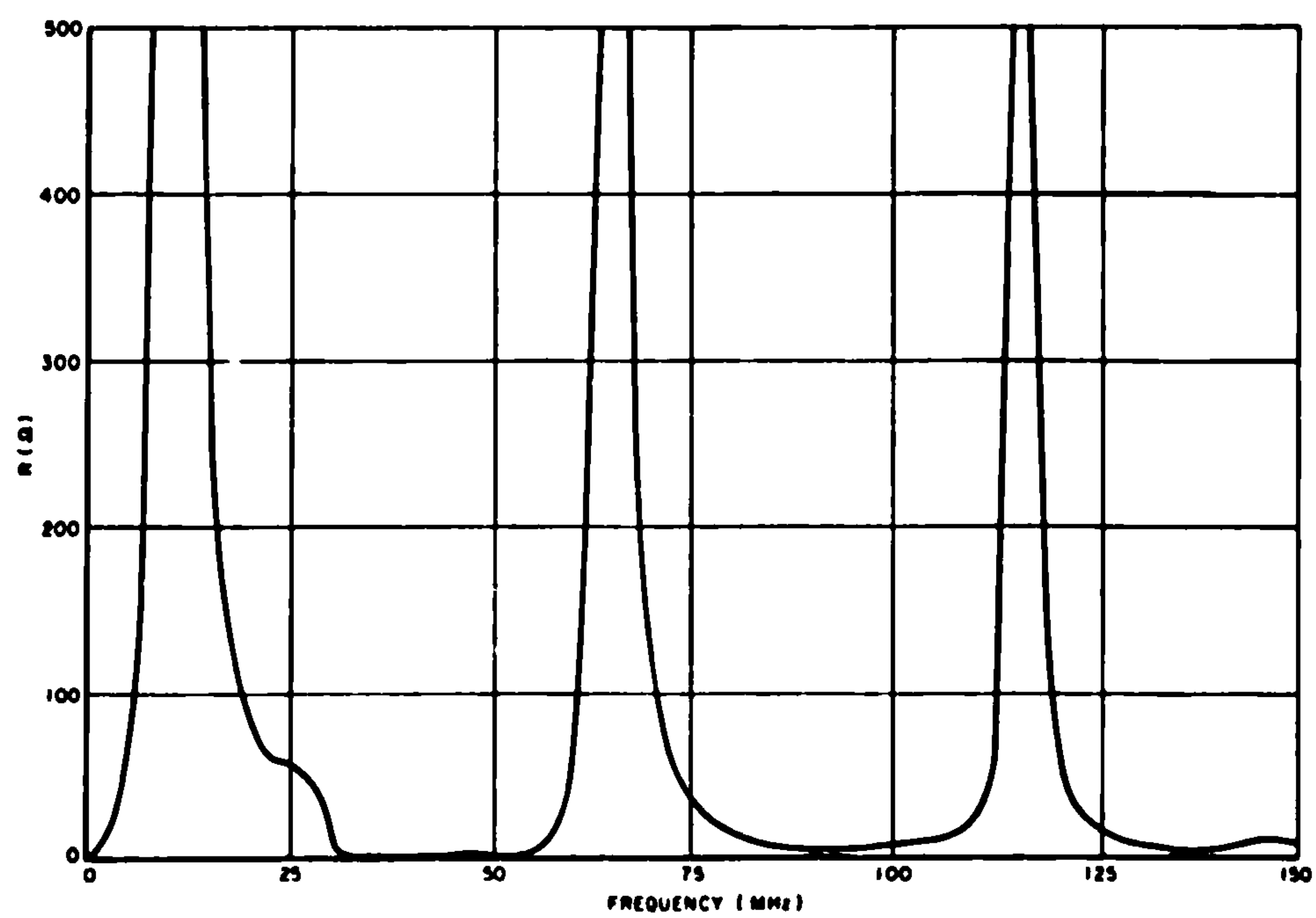

Fig. 8. Input resistance of a 5-turn loop (measured).

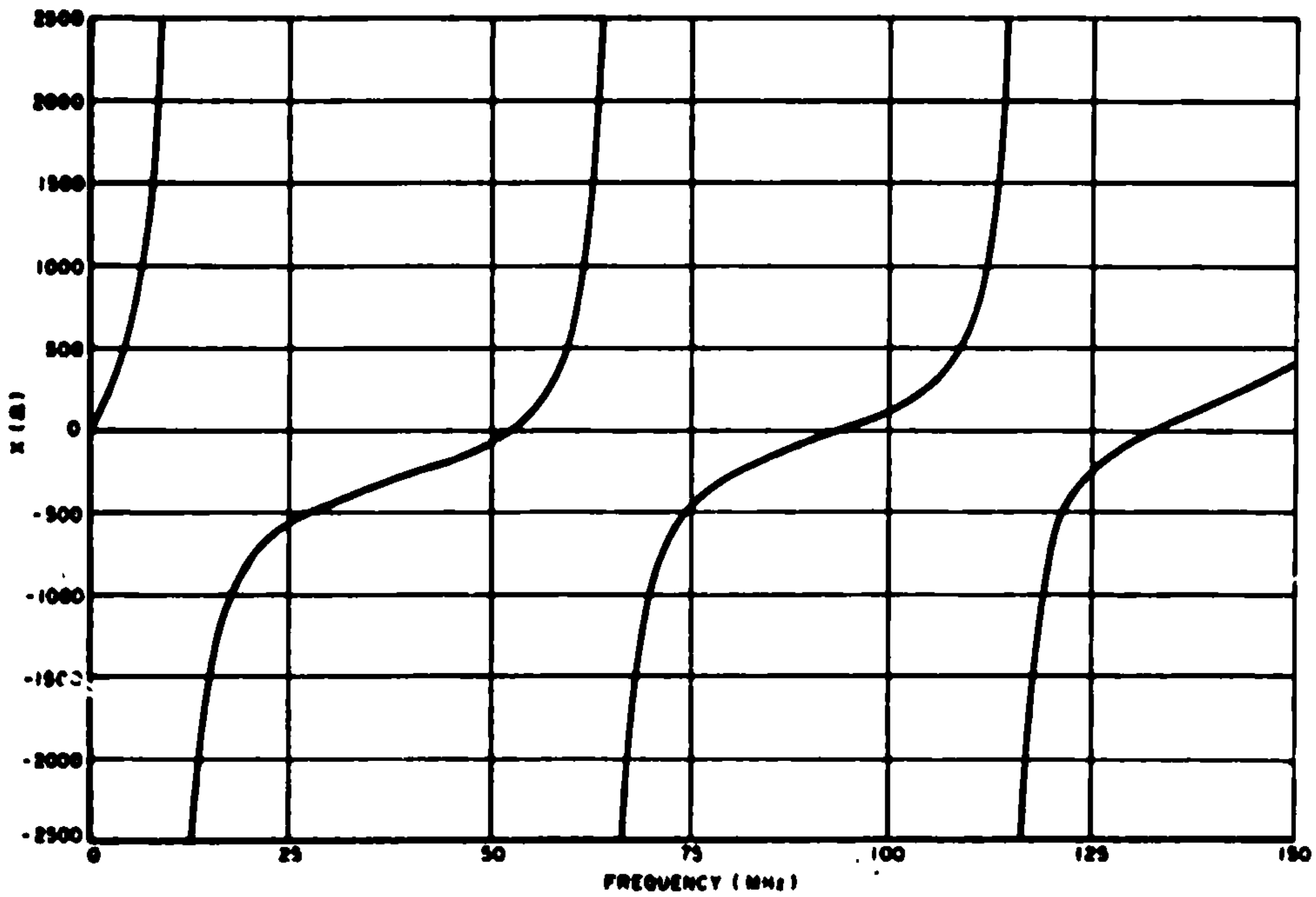

Fig. 9. Input reactance of a 5-turn loop (measured).

Fig. (8) is a plot of the input resistance and Fig. (9) depicts the input reactance.

A comparison of calculated and measured resistance is shown in Fig. (10). The results of Munk's analysis and the actual measured results agree quite well in form; however, the frequencies at which the resistance peaks actually occur are somewhat lower than the frequencies at which the theory predicts that the peaks should occur. This is probably due to the fact that the adjacent turns of the loop produce loading which decreases the phase velocity of the

traveling waves on the loop to a value which is less than the free space phase velocity.* It should be recalled that in Munk's approximate analysis it was assumed that the velocity of the traveling waves was that of free space. Thus the effect of loading would be to cause the structure to resonate at frequencies somewhat lower than those predicted by Munk's analysis.

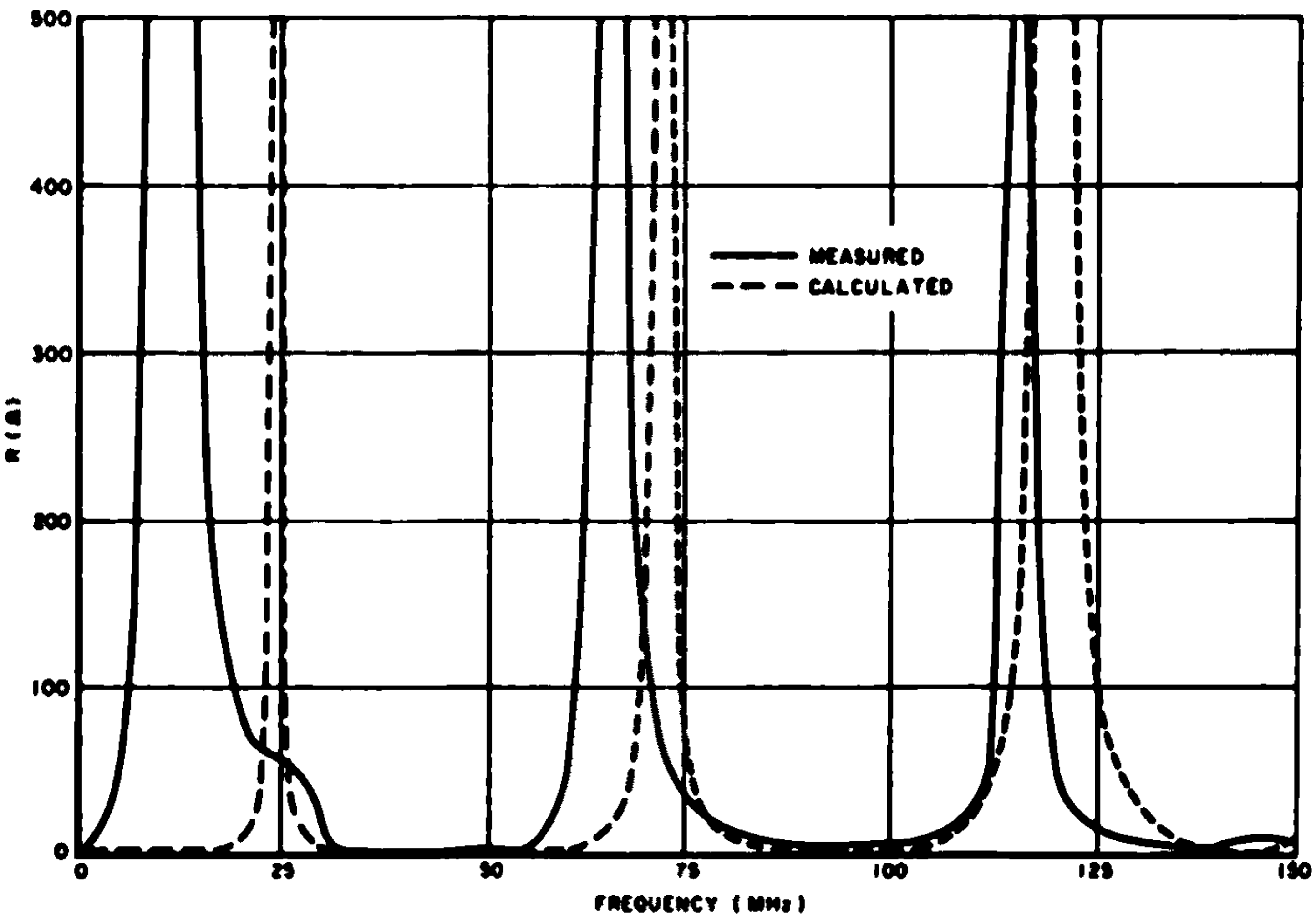

Fig. 10. Radiation resistance of a 5-turn loop.

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*The measured resonant frequencies indicated in Figs. (8) and (9) were verified by grid-dip meter measurements.

B. Efficiency Measurements

Efficiency measurements with the resistance comparison method have been made on the 5-turn copper loop antenna of section I-C by comparing its input resistance with that of a geometrically identical 5-turn brass loop. For copper and brass, Eq. (39) gives[13]

(44) $r = 1.90$

The measured values of efficiency are shown plotted in Fig. (11) with calculated values from Munk's analysis. As seen from Fig. (11), the measurements are somewhat scattered but tend to substantiate the predictions of Munk's analysis.

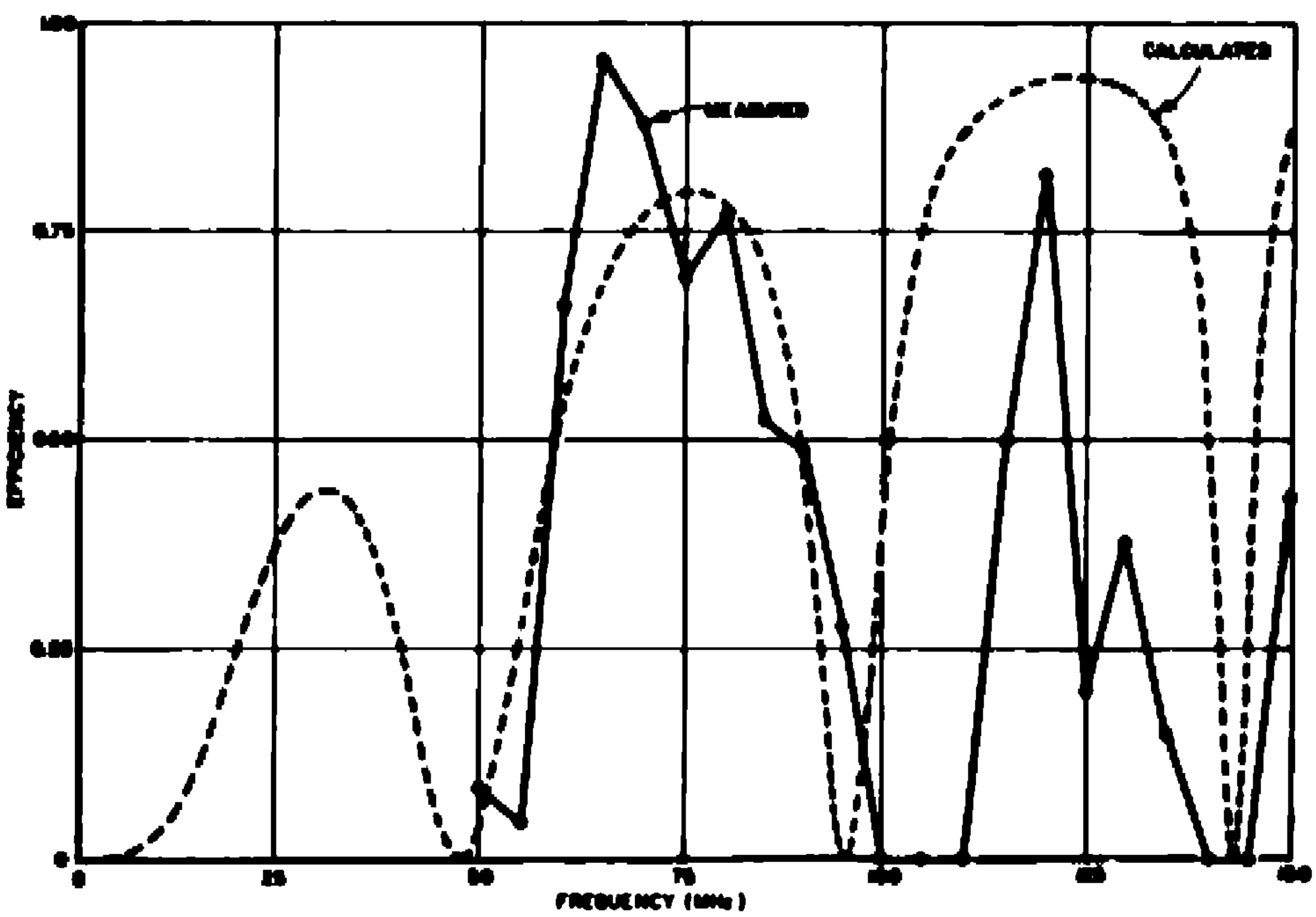


Fig. 11. Efficiency of a 5-turn loop.

The experimental errors from Eq. (43) have been plotted as a function of measured efficiency in Figs. (12) and (13) for the value of r given in Eq. (44). The observational errors Δr and $\Delta\left(\frac{R^{(2)}}{R^{(1)}}\right)$ are shown as parameters on the graphs. The observational error Δr was estimated from maximum and minimum values given in tables[18] to be about 8%. The error $\Delta\left(\frac{R^{(2)}}{R^{(1)}}\right)$ was estimated from variations in several sets of measurements to be about 7%. To find the maximum experimental error for any efficiency one simply adds the errors from Figs. (12) and (13). Since the actual value of $E^{(1)}$ cannot be known, the usual practice is to express the efficiency as the measured value plus and minus the error calculated from the measured value.[19]

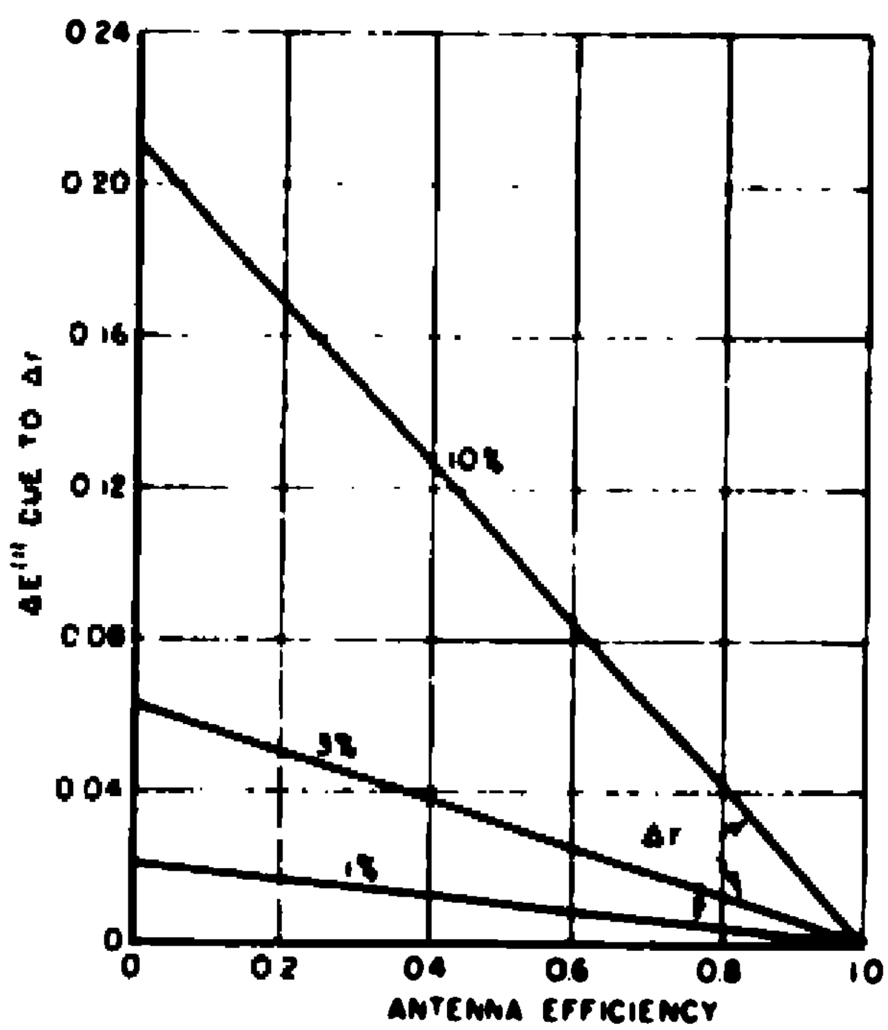


Fig. 12. Error in efficiency measurement due to error in r.

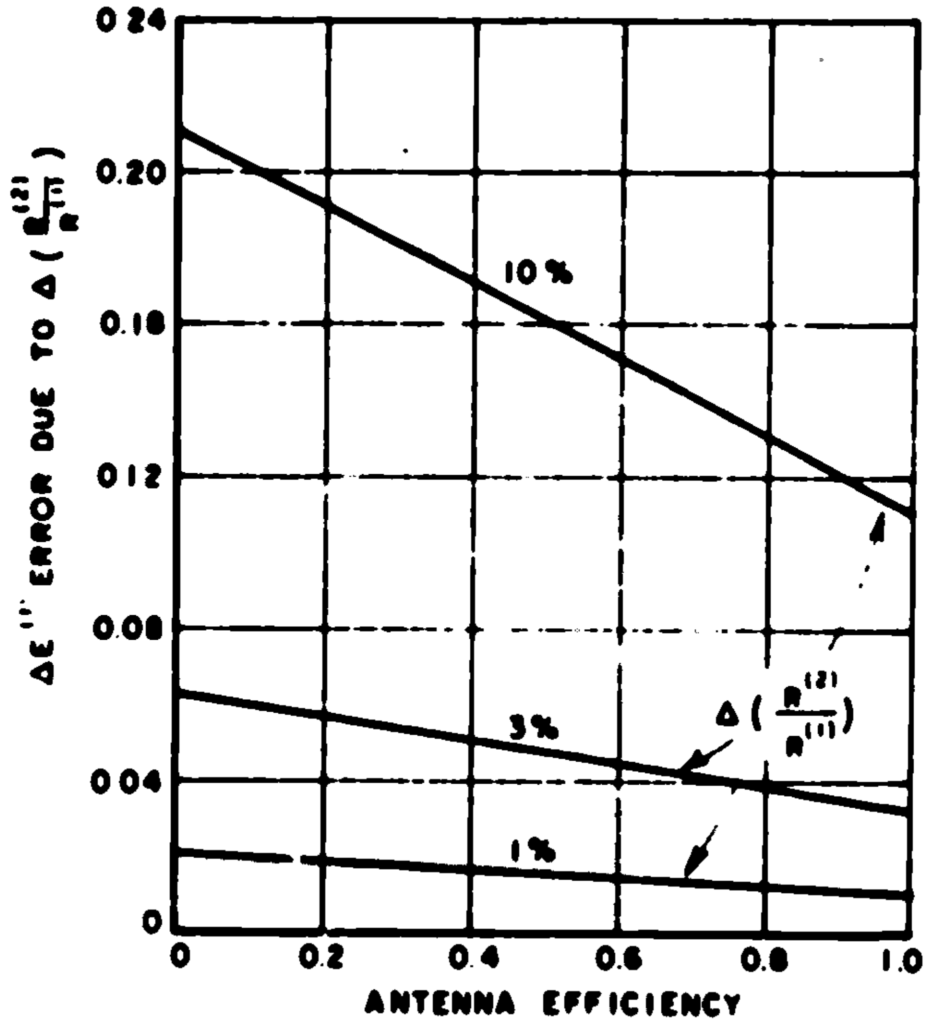


Fig. 13. Error in efficiency measurement due to error in $R^{(2)}/R^{(1)}$.

For example, with the measured value $E^{(1)} = 0.8$ and a Δr of about 8% , Fig. (12) shows an error of about 3%. With $E^{(1)} = 0.8$ and $\Delta\left(\frac{R^{(2)}}{R^{(1)}}\right)$ about 7%, Fig. (13) shows an error of 9% The efficiency is then $E^{(1)} = 0.80 \pm 0.12$. It should be noted that the error calculated in this way does not account for systematic errors in the measuring equipment.

In the manner described above Figs. (12) and (13) were used to determine the maximum and minimum values of efficiency corresponding to the measured values shown in Fig. (11). The maximum and minimum values of efficiency have been plotted in Fig. (14) and the area between the two curves has been shaded.

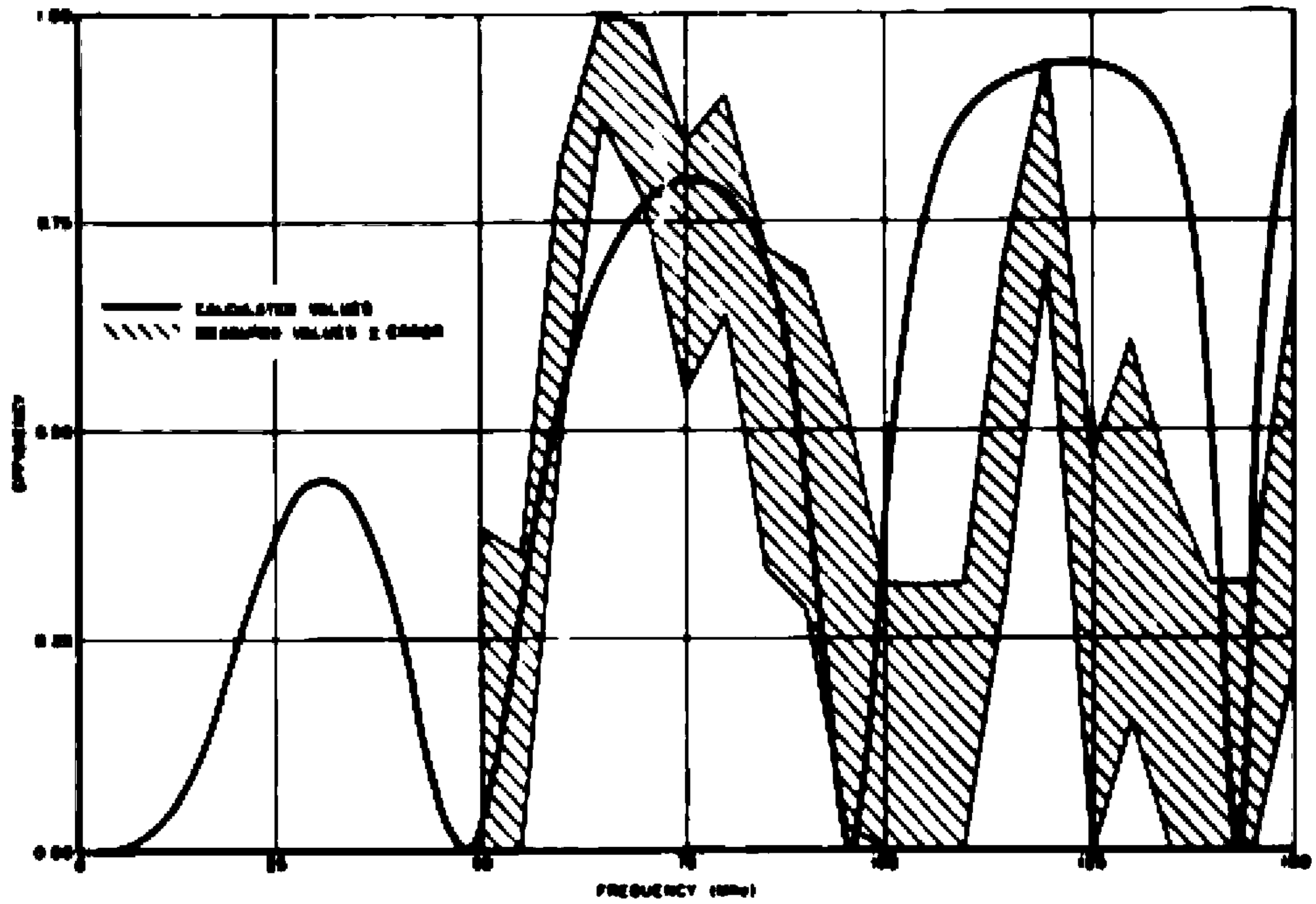


Fig. 14. Measured and calculated values of efficiency.

The calculated values of efficiency have also been plotted in Fig. (14) and are seen to be in fairly good agreement with the measured values.

CHAPTER V
SUMMARY AND CONCLUSIONS

In Chapter II the theoretical calculations of the input resistance and radiation efficiency of a multi-turn loop antenna have been reviewed. The theoretical input resistance and radiation efficiency of a particular loop were presented in Figs. (4) and (5).

In part A of Chapter III, techniques were discussed for measuring the impedance properties of an antenna. Results of the application of these techniques to the particular loop under study were presented in Chapter IV, Figs. (8) and (9).

In Part B of Chapter III several established techniques for the measurement of antenna radiation efficiency were discussed. Finally, in Chapter III a resistance-comparison method of measuring antenna efficiency was developed. It should be emphasized that the resistance comparison method is an adaptation of Crowley's method to antennas with a well defined set of terminals. A tacit assumption used in the development of the resistance comparison method is that the input resistance of the antenna can be adequately represented as a loss resistance in series with a radiation resistance. This is generally true for any antenna having a well defined set of terminals. The efficiency of the particular loop under study was measured by the

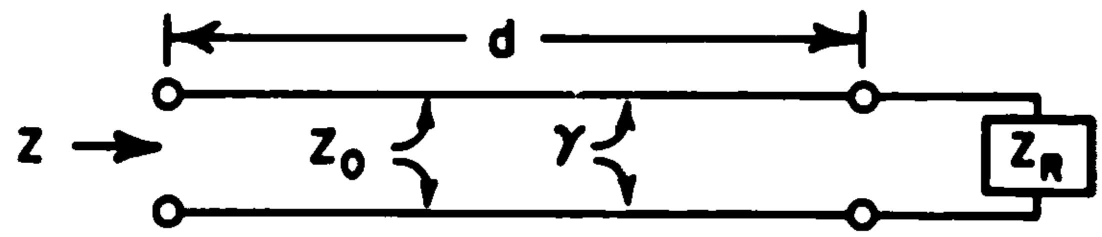
resistance-comparison method and the results presented in Chapter IV, Figs. (11) and (14).

Referring to Fig. (10) it is seen that the theory gives a good indication of the behavior of the input resistance in the range beyond the first resistance peak. In Figs. (11) and (14) it is seen that the measured values of efficiency, although somewhat scattered, do tend to substantiate the predictions of Munk's theory. It is therefore concluded that Munk's theory provides a good general description of a multi-turn loop antenna and that the resistance comparison method of measuring antenna efficiency is a practical method of measurement especially at frequencies below 100 MHz.

APPENDIX I

DETAILS OF THE METHOD OF EVALUATION OF PARAMETERS OF THE BALUN TRANSFORMER

Consider the transmission line circuit shown below.



The characteristic impedance is Z_o (complex in general); the propagation constant is $\gamma = \alpha + j\beta$; and the length is d. The line is terminated in complex impedance Z_R. The impedance Z observed at the end of the li e is given by Moore[11] as

$$Z = Z_o \frac{1+\Gamma_R e^{-2\gamma d}}{1-\Gamma_R e^{-2\gamma d}} \quad , \tag{1a}$$

where

$$\Gamma_R = \frac{Z_R - Z_o}{Z_R + Z_o} \tag{2a}$$

One may solve for Z_R in terms of Z, Z_o, and d by inserting Eq. (2a) into Eq. (1a) and rearranging the result. When this is done, the result obtained is

$$Z_R = Z_o \frac{1 + \Gamma_R' e^{+2\gamma d}}{1 - \Gamma_R' e^{+2\gamma d}} \quad , \tag{3a}$$

where

$$\Gamma_R' = \frac{Z - Z_o}{Z + Z_o} \quad . \tag{4a}$$

If for a transmission line of interest, Z_o and γ are unknown, a method of measuring them must be established. This can be done as follows. By inserting Eq. (4a) into Eq. (3a) and rearranging, the exponential term may be isolated. The result is

$$Z_o^2 + Z_o(Z_R - Z)\left(\frac{1 + e^{+2\gamma d}}{1 - e^{+2\gamma d}}\right) - ZZ_R = 0 \quad . \tag{5a}$$

Now, if load impedance Z_{R1} produces impedance Z_1 at the opposite end of the line and load impedance Z_{R2} produces impedance Z_2 at the opposite end of the line, then these quantities satisfy the equations

$$Z_o^2 + Z_o(Z_{R1} - Z_1)\left(\frac{1 + e^{+2\gamma d}}{1 - e^{+2\gamma d}}\right) - Z_1 Z_{R1} = 0 \tag{6a}$$

and

$$Z_o^2 + Z_o(Z_{R2} - Z_2)\left(\frac{1 + e^{+2\gamma d}}{1 - e^{+2\gamma d}}\right) - Z_2 Z_{R2} = 0 \quad . \tag{7a}$$

Now, by multiplying Eq. (6a) by $(Z_{R2} - Z_2)$ and Eq. (7a) by $(Z_{R1} - Z_1)$ and subtracting the results, one obtains

$$Z_o^2(Z_{R2} - Z_2 - Z_{R1} + Z_1) = Z_1 Z_{R1} Z_{R2} - Z_1 Z_2 Z_{R1} + Z_1 Z_2 Z_{R2} - Z_2 Z_{R1} Z_{R2} \quad , \tag{8a}$$

or

$$Z_o^2 = \frac{Z_1 Z_2 (Z_{R2} - Z_{R1}) + Z_{R1} Z_{R2}(Z_1 - Z_2)}{Z_{R2} - Z_{R1} + Z_1 - Z_2} \quad . \tag{9a}$$

Now if $Z_{R1} = 0$ (short circuit) produces Z^{sc} at the left end and $Z_{R2} = \infty$ (open circuit) produces Z^{oc} at the left end of the line then the characteristic impedance is

$$(10a) \qquad Z_o = \sqrt{Z^{oc}\, Z^{sc}} \quad ,$$

and from Eq. (1a) and Eq. (2a) (with $Z_R = 0$)

$$(11a) \qquad Z^{sc} = Z_o \quad \frac{1 - e^{-2\gamma d}}{1 + e^{-2\gamma d}} \quad .$$

Solving Eq. (11a) for the exponential term gives

$$(12a) \qquad e^{-2\gamma d} = \frac{Z_o - Z^{sc}}{Z_o + Z^{sc}} \quad ,$$

or

$$(13a) \qquad e^{-2\alpha d}\; e^{-2j\beta d} = \frac{Z_o - Z^{sc}}{Z_o + Z^{sc}} \quad .$$

Thus $2\alpha d$ and $2\beta d$ are evaluated as

$$(14a) \qquad 2\alpha d = \log_e \left| \frac{Z_o + Z^{sc}}{Z_o - Z^{sc}} \right| \quad ,$$

and

$$(15a) \qquad 2\beta d = -\tan^{-1} \left[I_M \left[\frac{Z_o - Z^{sc}}{Z_o + Z^{sc}}\right] \Big/ R_E \left[\frac{Z_o - Z^{sc}}{Z_o + Z^{sc}}\right] \right] \quad .$$

By observation, it was determined that the balun transformer (Anzac Model XT-617) used in making the measurements, behaved essentially as a transmission line in series with a 4 :1 impedance transformer. Thus multiplying the right-hand side of Eq. (3a) by 4 gives Eq. (18).

Short circuit and open circuit measurements were made on the balun transformer at frequencies between 50 and 150 MHz. The results were as follows.

F	Z^{sc}	Z^{oc}
50 MHz	$11.75 \angle +89.0°$	$223 \angle -76.0°$
75 MHz	$18.0 \angle +87.8°$	$142 \angle -79.6°$
100 MHz	$24.7 \angle +89.0°$	$103 \angle -82.0°$
125 MHz	$33.0 \angle +88.8°$	$78.5 \angle -82.5°$
150 MHz	$42.5 \angle +88.5°$	$61.1 \angle -82.2°$

From these values and Eq. (10a) the characteristic impedance at each frequency was calculated. The results were

F	Z_o (ohm)
50 MHz	50.8 + j6.69
75 MHz	50.5 + j3.60
100 MHz	50.3 + j3.10
125 MHz	50.8 + j2.80
150 MHz	50.9 + j2.80

For each value of Z_o and Z^{sc} given above, $2\alpha d$ and $2\beta d$ were calculated from Eqs. (14a) and (15a).

F	$2\alpha d$	$2\beta d$
50 MHz	0.0672	0.446
75 MHz	0.0725	0.678
100 MHz	0.0619	0.906
125 MHz	0.0683	1.148
150 MHz	0.0780	1.388

Eqs. (14a) and (15a) constitute a system of two equations and three unknowns (α, β, d). Thus one of these may be specified arbitrarily. It was decided to assume that the phase velocity of the transmission line was equal to that of free space. i.e.,

$$\beta = \frac{2\pi F_{MHz}}{300}$$

Under this assumption, the average length d and attenuation constant α were

$$d = 0.217 \quad \text{meters}$$

$$\alpha = 0.160 \quad (\text{meters})^{-1}$$

The average characteristic impedance was found to be

$$Z_o = 50.66 + j3.80 \quad \Omega \quad .$$

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